MEASURING THE ANGLE OF REPOSE OF GRANULAR SYSTEMS USING HOLLOW CYLINDERS

by

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The angle of repose of granular systems was investigated using hollow cylinders of different diameters and lengths and bases of different degree of roughness. The cylinders contained homogeneous samples of sand and gravel as well mixtures of sand and gravel with the sand either thoroughly mixed with the gravel or in layered systems. From the laboratory experiments, the following conclusions can be made: (a) the angle of repose of the granular systems was influenced by the degree of roughness of the base on which the grains came to rest. The rougher the base was, the higher was the angle of repose; (b) the mode of failure of the conical pile of grains was different depending if the base was rough or smooth. For a rough base, the failure took place on the face of the conical shape. For a smooth base, the failure took place at the base of the conical shape; (c) the lifting velocities of the cylinders were varied between a slow velocity (2 to 3 cm/sec) and a high velocity (7 to 8 cm/sec). The angle of repose was found to be smaller when the high velocity of cylinder lifting was used regardless of the bases' roughness; (d) the angle of repose was found to decrease in value as the amount of material contained in the cylinders increased in value; (e) the experiments on mixtures of sand and gravel indicated that their angle of repose decreased in value as the percentage of sand in the mixture increased in value; (f) the experiments on layered granular systems (gravel on top of sand) indicated that, regardless of the lifting velocity of the cylinders, the angle of repose of the layered systems decreased in value as the height of the sand layer in the composite increased in value; and (g) the results of the tests helped to explain the angle of repose found in real rock slopes.

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1.0 INTRODUCTION

Although many previous studies have been conducted to explain the behavior of granular materials, few of them have been concerned about the effects of non-homogeneous layered granular materials on the angle of repose.

This study, based on a series of laboratory experiments, primarily focuses on the angle of repose after the failure of granular deposits made of a single and two layers of granular materials (sand and gravel). The laboratory tests made of two layers will try to simulate the behavior of similar systems in the field and were designed to understand the influence of such factors as the velocity of failure, the arrangement of the layers, and the roughness of the base on which the granular materials were deposited.

Layered systems can occur in fluvial systems. It is known that sand and gravel are deposited in a fluvial bed. When the sand and gravel move to the bottom of the river, due to their different settling velocities, a process called sorting, is taking place. The settling velocity is function of the size and density of particles and is an indicator to show how quickly these transported particles by water flow can accumulate in the bed. During the low velocity flow of rivers, gravel will accumulate at the bottom of rivers, while sand will accumulate on top of the gravel. If the river changes its course, the gravel-sand deposit will remain in place. If this layered system experience failure, what will be the angle of repose of this system? This study again will try to give an answer to this question.

Layered granular materials also occur in the field as a result of crushing. This crushing takes place in the bottom section of a deposit of granular material. The overburden pressure at the bottom of a granular material could exceed the compressive strength of the grains. If this takes place, the grains at the bottom of the deposit will break into pieces that are smaller than the original granular pieces. The crushing will change the original deposit from one that is made of unbroken granular material to a deposit that is made of two different layers. The top layer will be made of unbroken pieces, and the lower bottom will be made of smaller pieces resulting from the fragmentation process. An example of this fragmentation takes place under glaciers. During the process of a huge glacier advance and retreat, the particles beneath it are subjected to great compressive loads. Some of the particles will break, some will not break. The particles that break forming small size particles that travel to the bottom of the deposit moving through the voids of the unbroken particles (pop-corn effect). After this takes place, the deposit will be made of a bottom layer of broken, smaller particles, and a top layer of large unbroken particles. If this layered granular deposit fails, what will be the angle of repose of the two layered system? This study would like to give an answer to this question.

Fig. 1 and 2 shows the evolution of breakage of granular materials when subjected to statically increasing compressive loads and centrifuge induced compressive loads respectively. The crushing simulation was conducted by Lobo-Guerrero and Vallejo (2010) using the Discrete Element Method (DEM). Under statically increasing compressive loads (Fig. 1), the crushed particles are uniformly distributed in the samples. For the case of compressive loads simulated by the centrifuge, the crushed particles are preferentially located at the bottom of the samples. If the systems shown in the field experience failure by the removal of the confining walls, what will be the angle of repose? This study would like to give an answer to this question.

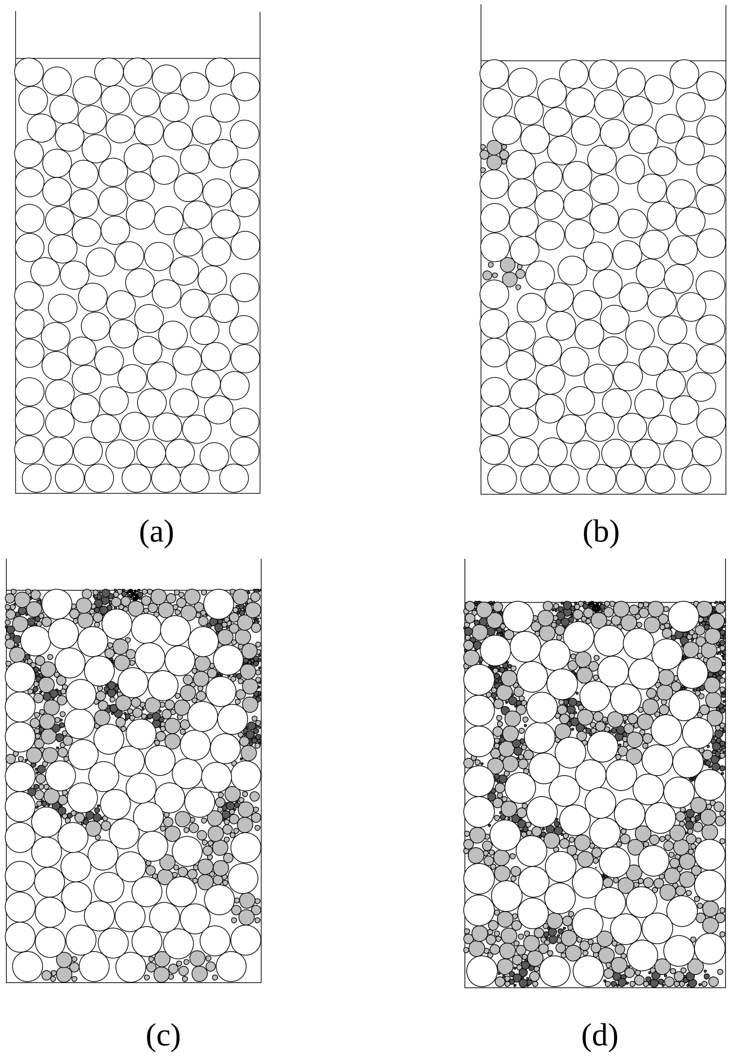


Figure 1 . Evolution of crushing under statically increasing one dimensional compressive loads: (a) 1×10^4 N (b) 3×10^4 N (c)7.5×10^4 N (d) 1×10^5 N (Lobo-Guerrero and Vallejo, 2010)

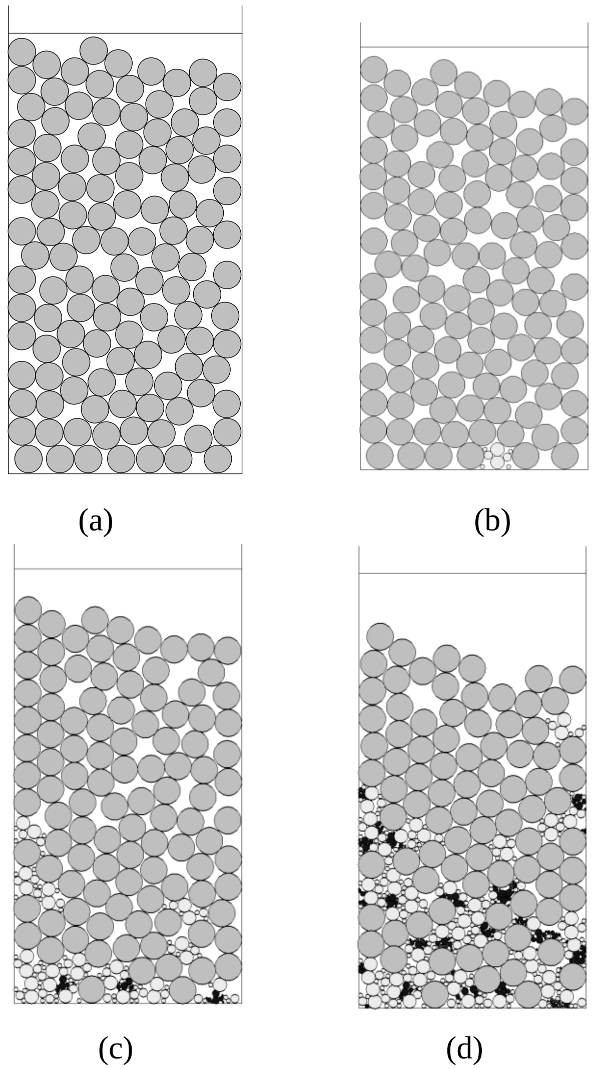


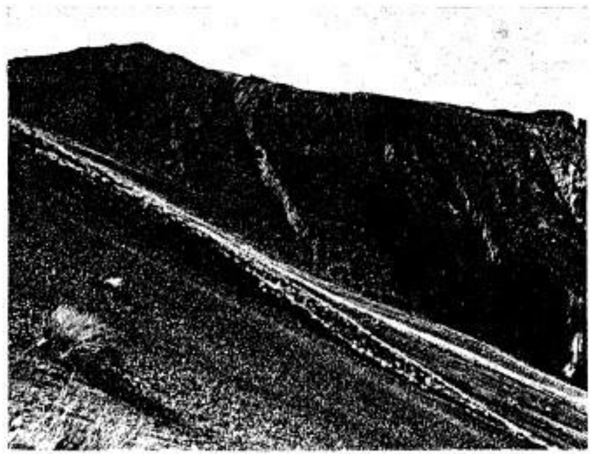
Figure 2. Crushing under simulated centrifuge pressure: (a) 1g, (b) 100g, (c) 600g, (d) 1800g (Lobo-Guerrero and Vallejo, 2010)

2.0 FORMATION OF LAYERED SYSTEMS IN THE FIELD

It is important to understand the form and configuration of layered slope deposits in nature because the layered system could affect both their physical and hydrological properties. The characteristics of deposits beneath the surface of a scree slope in Northern Craigieburn Range, New Zealand, were first studied by Pierson (1982). Before his studies, researchers of this area paid more attention to its characteristics landform and the geomorphic processes that caused the shape of the landform but without mentioning about the influence of the deposits beneath the surface on the geomorphology of the area. The area where the slope studied by Pierson is located is strongly influenced by glacial and fluvial erosion. Based on the large-scale geometry, the slope form was subdivided into three types by Pierson: (1) sheets; (2) cones and (3) gullies (Fig. 3). And also, the deposits beneath the slope surface, which is mantled by scree deposits, were subdivided into three types: (1) non-layered openwork gravels; (2) stratified gravels, sands and silts; (3) truncated silt-loam sub-soils (Fig. 4). The slope form types seemly determine which type of deposits beneath their surface, because not all deposits can be found in every each type of slopes. The openwork gravels layer can only be located in scree cones and scree sheets and the truncated silt-loam sbusoils deposit are associated with patchy scree sheets and stone pavements. Conversely, the well stratified deposit, type 2, is encountered with all three types of slopes.

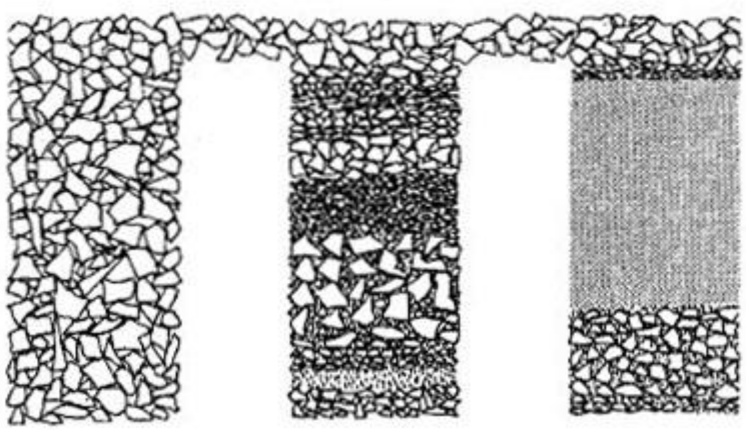
(a)

(b)



(c)

Figure 3. Three mainly basic forms of Craigieburn scree slopes:(a) scree sheets-island of vegetation in middle distance; (b) scree cone-fresh debris-flow deposits on scree surface; (c) scree gully-three example of erifying ages on far slope (dark horizontal bands are trial plantings of pine trees). (Pierson, 1982)



(a) (b) (c)

Figure 4. Schematic profiles of the three basic categories of slope deposits encountered on north Carigieburn scree slopes: (a) Openwork gravel. This is a non-stratified deposit of fairly angular fine, medium, or coarse gravel. Only a trace of fines is present, adhering to the coarse particles. (b) Stratified gravel and fines. In these deposits a thin compact layer of fines (a crust when dry) underlies the surface gravel. Beneath this are mixed layers (often with indistinct boundaries) of fine gravel, gravelly sand, and sometimes sand or silty sand. (c) Buried, truncated soil. Beneath the surface gravel and “crust” of fines as a well developed, truncated soil that is commonly the B horizon of a mature forest soil. It is characteristically developed in a discrete layer of silt or sandy silt that overlies stratified fine gravels typical of Type 2 deposit. (Pierson, 1982)

From this report, 70 percentages of the sites which were selected to study were well stratified deposits (Type 2 and Type 3) beneath the surface of a thin gravel layer. This gravel surface layer was commonly from 2 to 20 cm thick. Most of these gravel surface layers were unstable and the deposits underlying them formed a firm and smooth surface on which the gravels can move downward when they were triggered by forces. The slope angles in study area range from 20 degrees to 35 degrees. The granular slopes in New Zealand shown in Figs. 3 and 4 represent slopes that are layered and have reached after failure the angle of repose.

3.0 THEORETICAL ANALYSIS TO EXPLAIN THE DEGREE OF CRUSHING AND STRATIFICATION OF GRANULAR MATERIALS UNDER UNIAXIAL COMPRESSION

3.1 INTRODUCTION

In this study a theoretical analysis was developed to evaluate the crushing and stratification induced in homogeneous granular materials that were subjected to statically increasing compressive stress and centrifuge induced compressive tests. These tests were developed by Lobo-Guerrero and Vallejo (2010) and are presented in Figs. 1 and 2. These tests used the Discrete Element Method (DEM). In these tests, the samples were first generated in a simulated box container measuring 0.05 m in width and 0.10 m in height. This container was created by using the software PFC^{2D}, which is developed by the Itasca Consulting Group (2002). The coefficient of normal and shear stiffness of the walls of the container were set equal to $1x10^9$ N/M, and the coefficient of friction between the particles and the particles and the walls was set to be equal to 0.7. The radius of 120 particles randomly generated inside the box container was equal to 3 mm. The density of these particles was set to 2,500 kg/m^3. In the statically increasing uniaxial compression tests (Fig. 1), a moving piston plate with a velocity equal to 0.0625 mm/sec was used to apply compression to the particles. The compression from the moving piston plate varied from $1x10^4$ N to $1x10^5$ N. In simulated centrifuge induced compressive tests (Fig. 2), the gravity, G, applied on the particles was varied between 1g to 1800g (1g = 9.81 m/sec^2).

3.2 THE RESULTS OF THE CRUSHING TESTS AND THE ASSOCIATED THEORETICAL ANALYSIS

The main purpose of the theoretical analysis developed in this study is to develop a relationship that reflects the degree of crushing as a result of compression in the samples depicted by Figs. 1 and 2. Using this relationship, a prediction of the level of crushing in granular materials under any load will be obtained. The theoretical relationship will relate the percentage of particles that have been broken in function of the applied load. This theoretical relationship will also be helpful for our understanding how granular materials developed layers of different size particles.

Based on the results of the simulated compression tests (Fig. 1 and Fig.2), the number of particles under a certain load were counted manually. From the simulated compression tests under statically increasing compression stress tests (Fig. 1), the number of particles was equal to

120, 134, 562 and 769 when the compressive loads were equal to 1×10^4 N, 3×10^4 N, 7.5×10^4 N, and 1×10^5 N respectively. For the simulated centrifuge induced compression tests (Fig. 2), the number of particles was 120, 127, 239 and 477 when the gravity G used in the tests was equal to 1g, 100g, 800g and 1800g respectively. If N_0 is the number of the original particles before breakage, and N is the number of particles after breakage, Using Figs. 1 and 2 one can obtain the values of N and the ratio of N/N_0 related to the applied loads (Fig.1) and gravity. This has been done in Tables 1 and 2.

Table 1. Ratio N/No for the statically increasing uniaxial compressive stress tests

N(Number of Particles)	$\frac{N}{N_0}$	P(10^4N)
$N_1 = 120$	1	1.2
$N_2 = 134$	1.1	3.0
$N_3 = 562$	4.7	7.5
$N_4 = 769$	6.4	10

Table 2. Ratio N/No for compressive stress tests in the centrifuge

N(Number of Particles)	$\frac{N}{N_0}$	Value of the gravity, G
$N_1 = 120$	1	G_1 = 1g
$N_2 = 127$	1.06	G_2=600g
$N_3 = 239$	1.99	G_3=1000g
$N_4 = 477$	3.98	G_4=1800g

(1g = 9.81 m/sec^2).

To determine whether a relationship exists between N/N_0 and the load P as well as between N/N_0 and the number of g's applied that produce different levels of crushing in the samples the following analysis based on exponential functions is presented (von Bertalanffy, 1959). Let's assume that the increment in number of particles as a result of crushing, dN, is proportional to the original number of particles, N, multiplied by some increment of load dP, then,

$$dN = \lambda N dP \qquad (1)$$

Where λ is a constant.

After rearranging terms we obtain,

$$(dN/N) = \lambda dP \qquad (2)$$

The integration of Eq. (2) results in the following relationship,

$$\ln N = \lambda P + C \qquad (3)$$

If the number of particles is N_0 when P=0, from Eq. (3) we obtain

$$\ln N_0 = C \qquad (4)$$

Replacing Eq. (4) into (3) we obtain,

$$\ln (N/N_0) = \lambda P \qquad (5)$$

Eq. (5) is the result of taking natural logarithms of the following equation,

$$N/N_0 = e^{\lambda P} \qquad (6)$$

Eq. (6) is the exponential function that will be used to interpret the evolution of crushing in the simulated tests depicted in Figs. 1 and 2. For the case of the centrifuge forces induced compressive loads, P in Eq. (6) is replaced by the gravity intensity, G Table 2).

The best exponential fit curve to the measured values of N/N_0 and the values of P and G in Tables 2 and 3 provides the following relationships:

From the results of Table 1:

$$N/N_0 = 0.6728e^{0.2343P} \tag{7}$$

From Table 2:

$$N/N_0 = 0.847e^{0.0008G} \tag{8}$$

The values obtained using Eqs. 8 and 9 have been superimposed on the measured values in Figs. 5 and 6. The measured values and the theoretical results compare well. Thus, the theoretical relationships (Eqs. 8 and 9) can be used to predict future fragmentation in the simulations shown in Figs, 1 and 2.

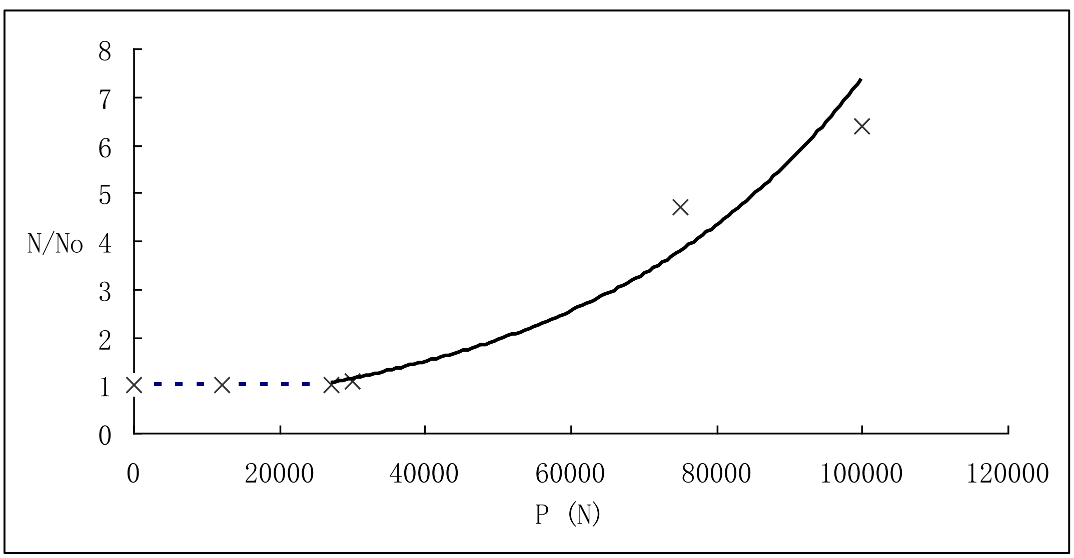


Figure 5. Uniaxial Compression Tests (full lines obtained using Eq. 7, X represents measurements from Table 1)

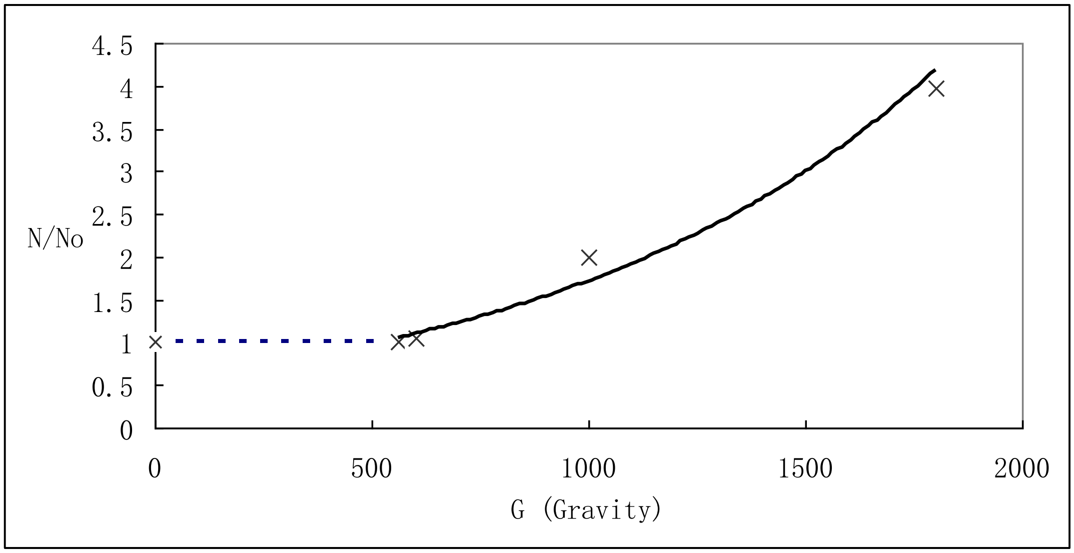


Figure 6. Centrifuge Induced Compression Tests (Full lines obtained using Eq. 8, and X is value measured and shown in Table 2)

4.0 ANGLE OF REPOSE OF NON-LAYERED-HOMOGENEOUS GRANULAR MATERIALS

4.1 INFINITE SLOPE STABILITY ANALYSIS

When a dry slope made of granular materials is subjected to failure, the granular materials accumulate at the toe of slope. The angle that the accumulated material forms with the horizontal is called the "angle of repose." This angle of repose has been found to be equal to the angle of internal friction, ϕ, of the granular material. This angle of repose can be measured in the laboratory using the funnel test. This test form part of the standard tests backed up by the American Society of Testing and Materials (ASTM) (Method No. C1444-00; ASTM, 2001). In the funnel test, granular material is placed in a funnel and then is slowly deposited on a horizontal surface as shown in Fig. 7. The granular material moves over the face of the accumulated triangular pile. When the movement stops, the inclination of the pile represents the angle of repose.

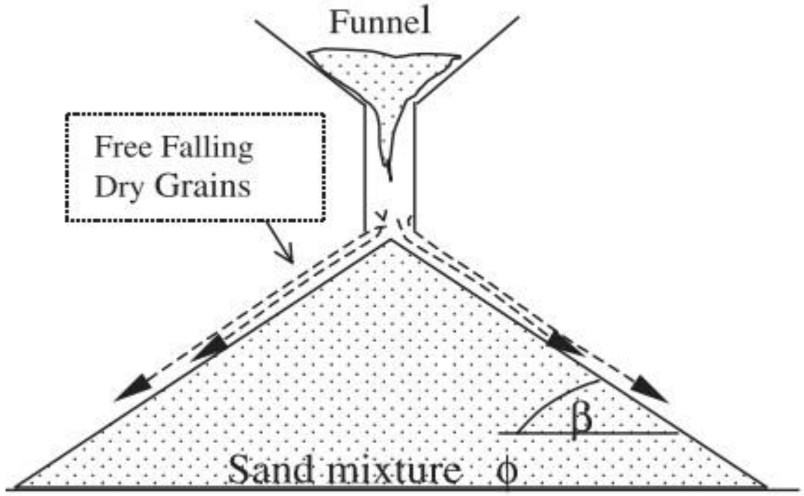


Figure 7. Free-falling Grain materials (sand) and the accumulated cone (Chik and Vallejo, 2004)

The relationship between the friction angle of the granular material and its angle of repose can be obtained using an infinite slope stability analysis (Das, 1985). For the analysis, the granular material is assumed to be dry.

Assuming the internal friction angle of the particles used in the infinite slope stability analysis is ϕ and the angle of the accumulated material is β. By using an infinite slope stability analysis as shown in Fig. 8, Das (1985) determined that at limit equilibrium conditions, the relationship between ϕ and β can be obtained from the following relationship:

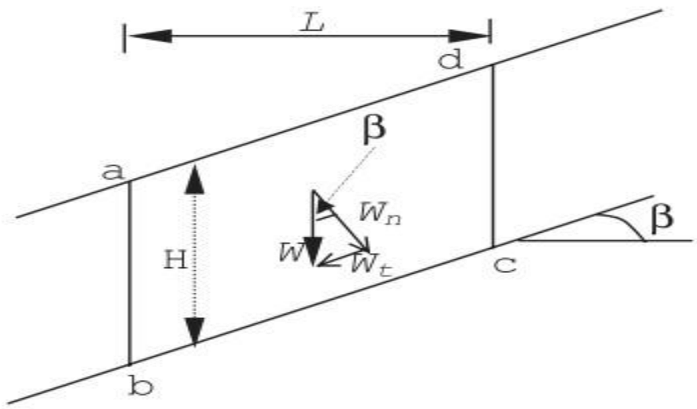


Figure 8. Parameters used in an infinite slope stability analysis [H = height of the sliding mass,β = the inclination of the failure surface and the ground surface, W = weight of the element abcd moving over the failure surface (Das, 1985)]

$$\tan \beta = \tan \phi \qquad (9)$$

Thus, the angle of repose, β , is equal to the angle of internal friction of the granular materials, ϕ .

4.2 SPREADING WEDGE SLOPE STABILITY ANALYSIS

Although many researchers have used funnel test (Fig. 7) to measure the angle of repose of granular materials, few of them discussed about the influence of the degree of roughness of the surface on which the granular material is deposited. If this surface is smooth, the interface friction angle, δ , between the granular material and the surface will be low ($\delta < \phi$, where ϕ is the friction angle of granular material, ϕ). Then the failure of a granular pile, will not take place as is shown in Fig. 7 (failure takes place on the face of the slope), but will take place as a "spreading type of failure" as is shown in Fig. 9. In the spreading type of failure, the material moves over the horizontal smooth surface, bc (Fig. 9). The movement of the material is caused by a lateral pressure P_{aH} that acts on a wedge bch located at the tip of a granular pile. The spreading of the granular material continuous as long as the force P_{aH} is bigger than the interface frictional force , $F\tau$, acting on the interface bc (Fig. 9. The spearding will stop when these two forces are equal. When these two forces are equal, the pile has acquired the angle of repose, When this β. At limit equilibrium conditions, Chik and Vallejo (2005) have developed an equation relating the interface friction angle, δ, the angle of repose, β, and the granular friction of the material, ϕ. This equation has the following form,

$$\tan\delta = \frac{\tan\beta\cos^2\phi}{2+2\sqrt{1-\left(\frac{\cos\phi}{\cos\beta}\right)^2}-\cos^2\phi} \qquad (10)$$

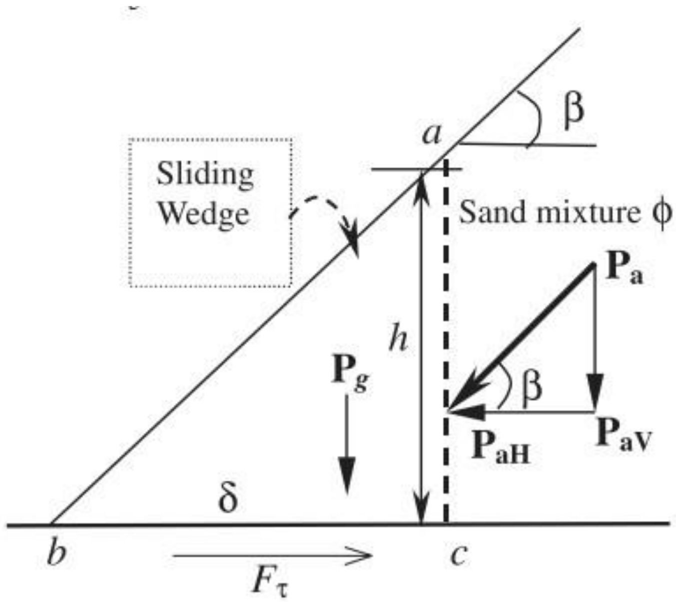


Figure 9. Sliding wedge used for the spreading type of slope stability analysis

Using Eq. 10, one can obtain the angle of repose, β , if one knows the angle of friction of the granular material, ϕ , and the angle interface friction angle, δ , between the granular material and the surface on which it spreads.

In their study, Chik and Vallejo (2005) measured the interface friction angle between granular materials and a smooth base (glass plate) and a rough base (porous stone). The angle δ was measured by putting sand in a metal ring that rested on either the glass or the porous stone. The assembly sat on a tilting table. Then, the tilting table was raised slowly until the ring and the granular material moved. When this took place, the angle of the tilting table represented the interface friction angle, δ, between the sand and the base used ($\delta = \theta$) (Fig. 10).

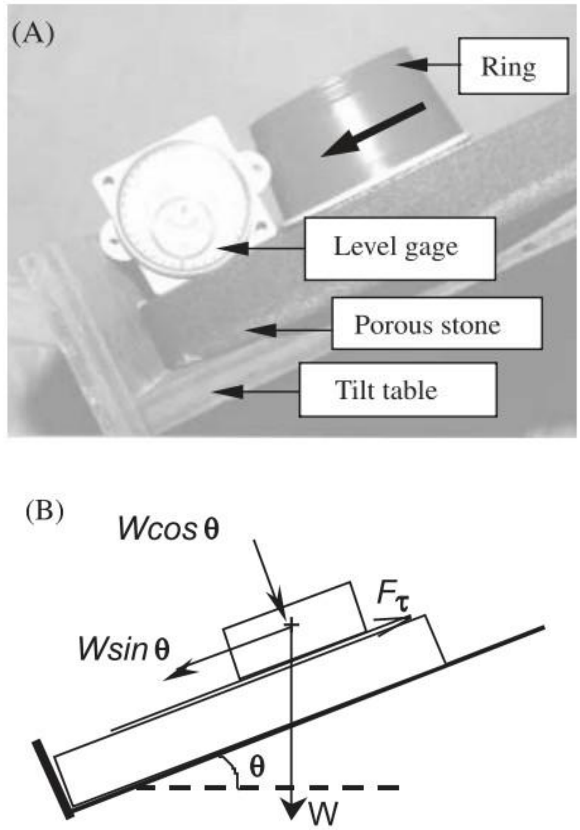


Figure 10. (A) The setup of measurement of interface friction angle using an adjustable inclined bench (B) Schematic diagram of forces acting on the inclined bench, θ , is the inclination angle (Chik and Vallejo, 2005).

The interface friction angle between the smooth and rough bases and mixtures of fine and coarse sand are shown in Fig. 11.

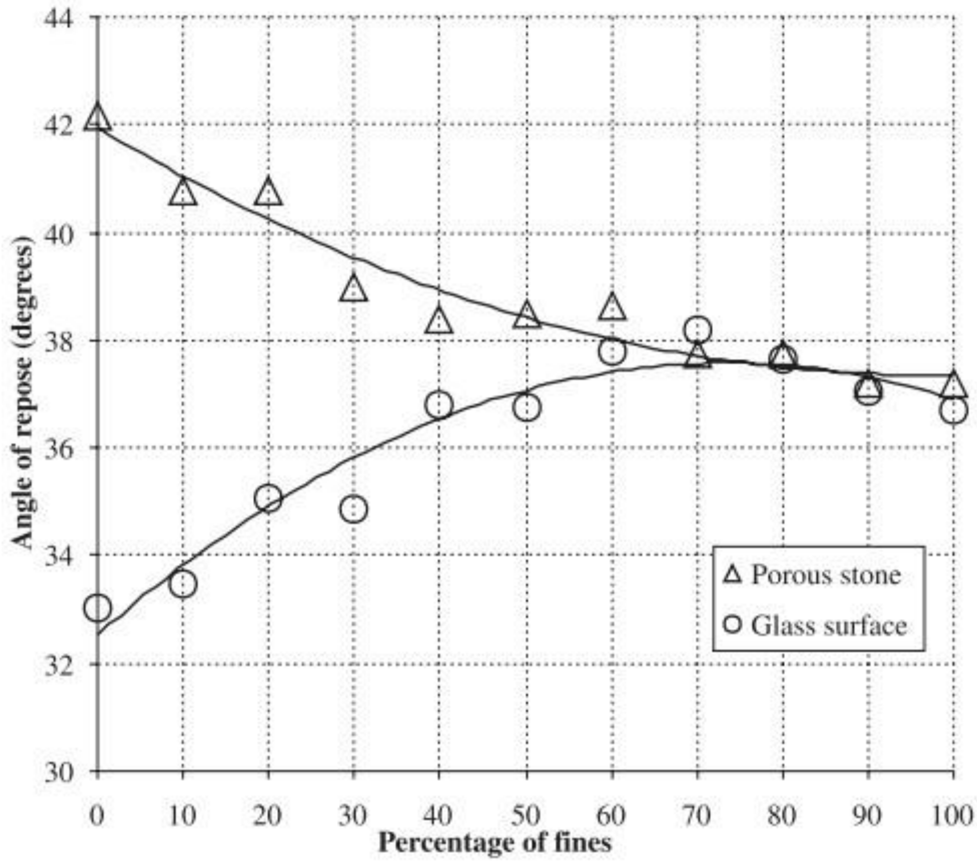


Figure 11. Angle of repose for a mixture coarse-fine sand on smooth and rough bases (glass plate and porous stone) (Chik and Vallejo, 2005)

4.3 GRANULAR SPREADING FROM THE LIFTING OF A CYLINDER WITH GRANULAR MATERIAL

Slopes can fail either very rapidly or slowly. Fig. 12 shows a sandstone slope in Utah that is made of a steep intact face and accumulated material at its toe. The accumulated material is the result of either rapid failure of a section of the steep face or a slow failre due to the physical weathering of the slope (i.e. freezing and thawing, heating and cooling). As a result of a failure of the steep face, the material accumulates at the toe of the slope. The accumulated material is usually granular debris that forms an angle with the base of the slope. This angle is the angle of repose.

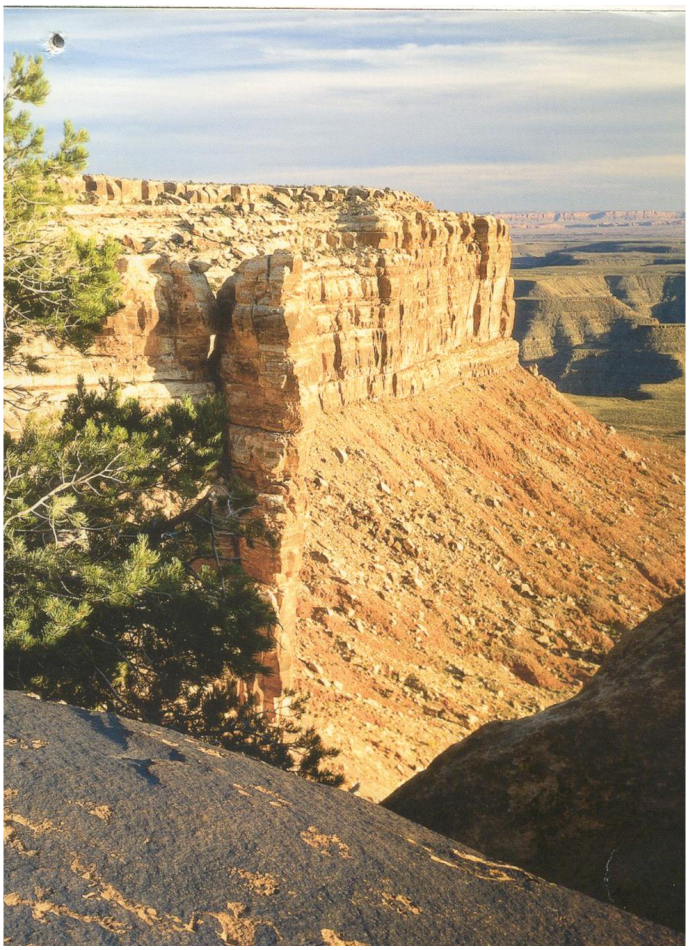


Figure 12. A sandstone slope in Utah formed of a steep face and an accumulated material at the toe of the slope

A variation of the funnel test discussed before to evaluate the angle of repose of granular materials has been developed by Lajeunesse, et al. (2004). In the test conducted by Lajeunesse et al., granular material is placed in a hollow cylinder when in contact with a base. Then the hollow cylinder is lifted and the granular material flows on the surface where before the cylinder was resting. As the cylinder is lifted, the granular material forms a pile with an external face inclined at the angle of repose (Fig. 13).

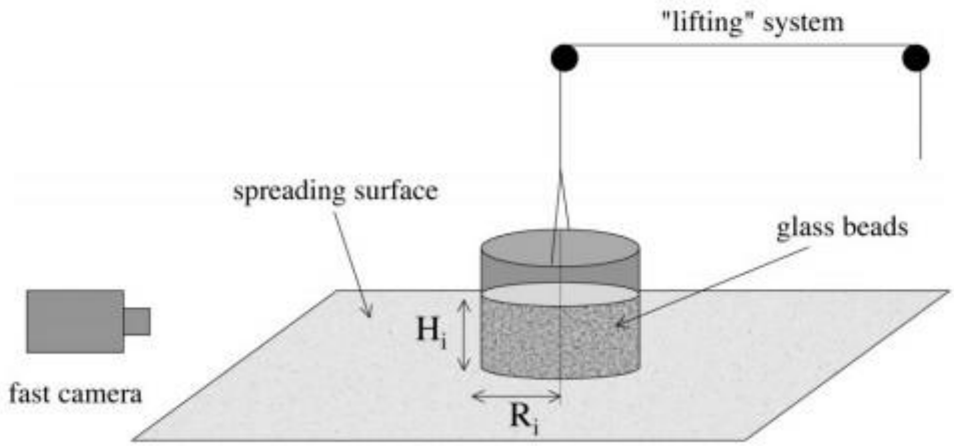


Figure 13. Scheme of the experimental setup to investigate the flowing behavior of granular materials on horizontal plate (Lajeunesse, et al. 2004)

For the laboratory experiments, Lajeunesse et al. (2004) used different type of materials and surfaces over which the materials flowed. Table 3 shows the properties of the granular materials and surfaces used. This experimental setup shown in Fig. 13 was conducted by Lajeunesse to study the shape of the resulting cone after the particles being released and flowing on the surface by lifting the cylinder. From the final shape the angle of repose was obtained.

Fig. 14 shows the flow of a granular mass (950 g) made of grains with diameter d = 350μm placed in a cylinder with internal diameter equal to 70. 5 mm. The base on which the material flowed is made of a smooth wooden surface. Fig. 14 shows the formation of the pile at different times after the starting of the test. The bottom section of Fig. 14 shows the final pile of granular material that formed at a time equal to 0.54s after the initiation of the test. The angle of repose of the granular material was found to be equal to 20 degrees.

Table 3. The diameter of the particles, d, and the type of the substrate used in the experiment conducted by Lajeunesse, et al. (2004)

Series	d	Substrate
1	350 ± 50 μm	Sandpaper of roughness $\lambda\approx540$ μm
2	350 ± 50 μm	Erodible bed (same granular material) of thickness 4 or 12 mm
3	350 ± 50 μm	Smooth wooden surface
4	1150 ± 150 μm	Sandpaper of roughness $\lambda\approx540$ μm

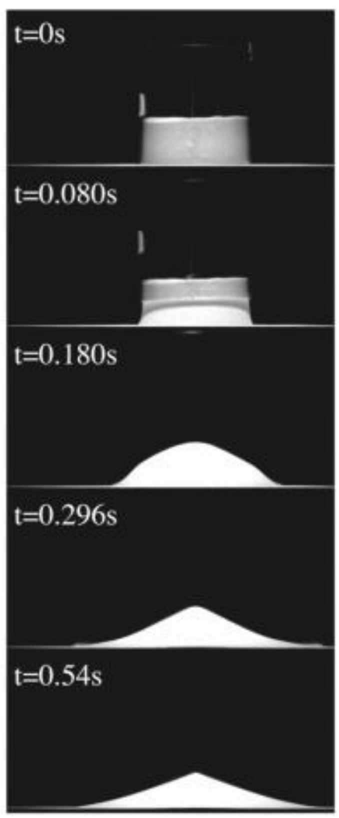


Figure 14. Flowing of granular material when a cylinder containing it is lifted (Lajeunesse,et al., 2004)

In the tests conducted by Lajeunesse, et al., (2004), the mass of granular material used was a homogeneous one. Also, the influence of the velocity of lifting the cylinder on the angle of repose of granular materials was not investigated. The lifting of the cylinder at different velocities will allow the granular materials to be deposited at different velocities. These different velocities of lifting of the cylinder represent the different velocities at which a material can fail from the steep face of the sandstone slope shown in Fig. 13. In this study, cylinders similar to the one shown in Figs. 13 and 14 were used to measure the angle of repose of granular materials in the form of sand and gravel. The tests were conducted using homogeneous samples of either sand or gravel, or a mixture of the two forming two layers. The influence of the velocity of lifting of the cylinders on the angle of repose was also investigated.

5.0 DESCRIPTION OF THE EQUIPMENT USED FOR THE LABORATORY TESTING PROGRAM

5.1 GRAVEL AND SAND USED IN EXPERIMENTS

In these experiments, gravel and fine sand were used in the experiments. The gravel has an average diameter d_{50} = 5 mm, and the sand used was Ottawa sand with an average diameter d_{50} = 0.59 mm. The specific gravity of the gravel, G_s, was equal to 2.6, and for the sand was equal to 2.65. The angle of internal friction for the sand was equal to 27 degrees. The angle of internal friction for the gravel was equal to 34 degrees. These angles were obtained by a funnel test and a rough base where the granular materials were deposited.

5.2 THE BASES INVOLVED IN EXPERIMENTS AND THE INTERFACE FRICTION ANGLE

In order to simulate the smooth and rough surfaces on which granular materials could be deposited, a glass plate was used as the smooth surface, and a porous stone and a wooden table was used as the rough surface. Fig. 15 shows these bases with gravel being deposited on their surfaces. Also, the angle of interface friction between the smooth and rough surfaces and the granular materials (sand and gravel) used in the experiments were measured in the laboratory. The equipment used for the interface friction angle experiments is similar to the one described in Section 4.2 of this thesis (Fig. 10), and the procedure used follows that of Chik and Vallejo, (2005).

(a)

(b)

(c)

Figure 15. Different bases used: (a) Glass Plate, (b) Porous Stone, and (c) Wooden table

The interface friction angles measured between granular materials used and the different bases are shown in Table 4. Table 4 indicates that the interface friction angle, δ , was influenced by the roughness of the base in contact with the granular materials. In general, it was found the rougher the surface is, the larger is the interface friction angle.

Table 4**.** Interface friction angle between granular particles and different bases

Grain Type	Base Type	Interface Friction Angle, δ (degrees)
Sand	Porous Stone	25
Sand	Glass Plate	16
Gravel	Porous Stone	26
Gravel	Glass Plate	15
Gravel	Wooden table	22

5.3 THE CYLINDERS USED AND THE GENERAL SETUP FOR THE LABORATORY EXPERIMENTS

In these experiments, particles in the form of sand, gravel or a mixture of the two in the form of layers were placed in plexiglass cylinders. One cylinder has an internal diameter equal to 5 cm and a height equal to 17.15 cm [Fig. 16(a)], , the second cylinder had an internal diameter equal to 12 cm and a height equal to 17.15 cm [Fig. 16 (b)], and the third cylinder had a diameter equal to 12.5 cm and a height equal to 108.6 cm[Fig. 16(c)]. The thickness of the cylinders was equal to 5 mm. The cylinders with the granular materials were placed on top of the three different surfaces. After this was accomplished, the cylinders were lifted manually at different velocities. The lifting involved two operators, one lifted the cylinders and the granular material spread over the selected surfaces, the other measured the velocity of lifting using a stop watch. Two different lifting velocities of the cylinders were used in the experiments. One was a slow velocity (2 to 3 cm/sec) that simulated the slow release of granular material from the top of a rock slope and its accumulation at its toe (Fig. 12). The other was a high velocity (7 to 8 cm/sec) that simulated the high release of granular material from the top of a rock slope and its accumulation at its toe (Fig. 12). The experiments were conducted using in the cylinders sand alone, gravel alone, or gravel and sand. When gravel and sand were used, one set of experiments used a mixture of sand and gravel, and the other set of experiments used the gravel in sand in layers. In the layered experiments, the sand layer was located at the bottom of the cylinder, with the gravel being located at the top. The sand at the bottom simulated the smaller material that usually accumulates at the bottom of a granular material and is the result of crushing [Fig. 2(d)].

(a)

(b)

(c)

Figure 16. The three different cylinders used in the experimental program

After the material was released from the cylinder, the angle of repose of the accumulated granular material was measured. The angle of repose was the average of four measurements at different locations in granular cone that formed after the lifting of the cylinders.

6.0 RESULTS FROM LABORATORY EXPERIMENTS

The experiments on the angle of repose formed when cylinders with granular material are lifted on top of surfaces with varying degree of roughness include: (a) cylinders with sand, (b) cylinders with gravel, and (c) cylinders with gravel and sand. When sand and gravel were used, one set of experiments used a mixture of the sand and gravel; the other set of experiments used the gravel and sand in layers. Next, the results of these experiments are described.

6.1 EXPERIMENTS WITH CYLINDER CONTAINING ONLY SAND

The cylinder with a 5 cm interior diameter and 17.15 cm in length [Fig. 16 (a)] was used for the lifting-spreading experiments. Two different lifting velocities were used, one was a slow velocity (2 cm/sec), and the other was a fast velocity (7 cm/sec). The surfaces over which the release of the sand took place were a smooth, glass plate [Fig. 15(a)], and a rough, porous stone [Fig. 15(b)]. The results of the experiments are shown in Tables 5 and 6.

Table 5. Cylinder experiments with sand, slow velocity, about 2.0 cm/sec

Percentage of sand by weight	Angle of repose on smooth base (degrees)	Angle of repose on rough base (degrees)
100	25.0	27.0

Table 6. Cylinder experiments with sand, high velocity, about 7.0 cm/sec

Percentage of sand by weight	Angle of repose on smooth base (degrees)	Angle of repose on rough base (degrees)
100	21.6	24.3

The results of the experiments indicate that the roughness of the base influenced the angle of repose. This being higher when the sand was released on the porous stone. Also, the lifting velocity of the cylinders influences the angle of repose. When the lifting was slower (2 cm/sec), the angle of repose was higher than that measured when the lifting velocity was faster (7 cm/sec). This later result took place regardless of the degree of roughness of the base on which the spreading of the sand took place (Tables 5 and 6).

6.2 EXPERIMENTS WITH CYLINDERS CONTAINING ONLY GRAVEL

For the experiments with gravel, the three cylinders (internal diameters equal to 5, 12 and 12.5 cm) shown in Fig. 16 were used. The results of the tests are shown in Tables 7 and 8. Table 7 shows the results of the angle of repose when the 5 cm cylinder was used. The gravel filled the total height of the cylinder (17.15 cm). The surfaces on which spreading took place were the smooth surface (glass surface), and the rough surface (porous stone). Two different velocities were used for the lifting of the cylinder. These were equal to 2 cm/sec and 7 cm/sec. The results of Tables 7 and 8 indicates that the angle of repose were smaller when the spreading took place on the smooth surface. Also, the higher the velocity of lifting of the cylinder, the smaller was the angle of repose.

Table 7. Cylinder experiments (5cm inner diameter) with gravel, slow velocity, about 2.0cm/sec

Percentage of gravel by weight	Angle of repose on smooth base (degrees)	Angle of repose on rough base (degrees)
100	28.0	33.5

Table 8. Cylinder experiments (5cm inner diameter) with gravel, slow velocity, about 7.0cm/sec

Percentage of gravel by weight	Angle of repose on smooth base (degrees)	Angle of repose on rough base (degrees)
100	25.0	32.0

The cylinder with 12 cm in diameter and 17.15 cm in height as well as the cylinder with 12.5 cm in diameter and 108.c cm in height were also used for the measurement of the angle of repose when they were filled with gravel.

The experiments using the cylinder measuring 12 cm in diameter are shown in Figures 17, 18 and 19. The cylinder was filled with gravel at three different heights. These heights were equal to 5.8 cm, 8.6 cm, and 17.15 cm. For this heights, the weight of the gravel in the cylinders were equal to 12.37, 21.88, and 34.21 N respectively. The angle of repose on surfaces of different degree of roughness and using different lifting velocities of the cylinders are shown in Table 9 and Figs. 17 to 19. The results indicate that the higher was the level of gravel in the cylinder; the lower was the angle of repose, regardless of the roughness of the surfaces used and the lifting velocities. Thus, the weight of the material released seems to affect the value of the angle of repose.

Table 9. Cylinder experiments (12.0cm inner diameter) with gravel, both slow velocity (about 3.0cm/sec) and high velocity (about 8.0cm/sec)

Weight of gravel (N)	Angle of repose on smooth base, slow velocity (degrees)	Angle of repose on rough base, High velocity (degrees)
12.37	28.6	22.1
21.88	25.0	19.0
34.21	23.0	16.9

Figure 17. Cylinder experiment (12.0cm inner diameter) with gravel (12.37N), slow velocity

Figure 18. Cylinder experiment (12.0cm inner diameter) with gravel (21.88N), slow velocity

Figure 19. Cylinder experiment (12.0cm inner diameter) with gravel (34.21N), slow velocity

The angle of repose experiments using gravel were also conducted on the large cylinder that measured 12.5 cm in diameter and 108.6 cm in height. The heights of gravel in the cylinder were equal to 20, 40 and 60 cm which corresponded to a weight of about 45.73, 92.87 and 142.34 N respectively. The base used for the experiments was the wooden table base. The lifting velocity was slow (2.5 cm/sec). The results of the experiments are shown in Table 10 and Figs. 20 to 22.

Table 10. Cylinder experiments (12.5cm inner diameter) with gravel, on wood table with about 2.5cm/sec lifting velocity

Weight of gravel (N)	Angle of repose on wood table (degrees)
45.73	25.0
92.87	16.0
142.34	11.9

Figure 20. Cylinder experiment (12.5cm inner diameter) with gravel (45.73N) on wood table

Figure 21. Cylinder experiment (12.5cm inner diameter) with gravel (92.87N) on wood table

Figure 22. Cylinder experiment (12.5cm inner diameter) with gravel (142.34N) on wood table

An analysis of the results shown in Table 10 and Figs. 20 to 22 indicates that the higher was the level of gravel in the cylinder; the lower was the angle of repose. Thus, the weight of the material released from the cylinders seems to affect the value of the angle of repose measured in the experiments.

When the gravel experiments were conducted using the bigger cylinders (12.0 cm and 12.5 cm inner diameters), the rough base (porous stone) and the smooth base (glass base) were not used because these bases were too small to hold all the gravel material released by the cylinders. For this reason, the surface of a big wooden table was used for the experiments (Fig. 20 to 22).

6.3 EXPERIMENTS USING CYLINDERS CONTAINING A MIXTURE OF SAND AND GRAVEL

Experiments using mixtures of sand and gravel were conducted using the small cylinder (5 cm in diameter). Mixtures with different proportions of sand and gravel filled the height of the cylinder (17.15 cm). The different mixtures were released on top of the smooth base (glass plate) and rough base (porous stone) at velocities equal to 2 cm/sec and 7 cm/sec (Table 11 and 12).

Table 11. Cylinder experiments using mixtures with slow lifting velocity (about 2.0cm/sec)

Percentage of sand by weight	Angle of repose on smooth base (degrees)	Angle of repose on rough base (degrees)
10	20.0	29.5
20	20.7	26.9
30	18.5	23.0
40	22.9	28.5
50	25.1	25.9
60	23.5	27.1
70	24.0	25.9
80	26.5	28.6
90	26.1	25.9

Table 12. Cylinder experiments using mixtures and high lifting velocity (about 7.0cm/sec)

Percentage of sand by weight	Angle of repose on smooth base (degrees)	Angle of repose on rough base (degrees)
10	17.5	30.5
20	20	24.5
30	18	23.5
40	21	27
50	21.5	25
60	22	26.5
70	26	23.5
80	22.9	24
90	23.5	25.1

An analysis of the results of Tables 11 and 12 indicates that the angle of repose was influenced by the degree of roughness of the bases used and the lifting velocities of the cylinders. In general, the results of Tables 11 and 12 indicate that the angle of repose was higher when the porous stone (rough base) was used. Also, the higher was the lifting velocity of the cylinders; the lower was the angle of repose, especially for the tests that used the smooth base. Further analysis will be conducted in Section 7 of this Thesis entitled: Analysis and Application of the Laboratory Results.

6.4 EXPERIMENTS USING A CYLINDER THAT CONTAINS A LAYERED SYSTEM OF GRAVEL AND SAND

The layered system used in experiments was set up with the 5.0 cm inner diameter cylinder, place on both the smooth and rough bases. The sand was placed at the bottom of the cylinder with the gravel on top in order to simulate conditions such as those shown in Figs. 2(d) and 4(b). Two different lifting velocities were used. One was a slow (2.5 cm/sec) and the other was fast (7 cm/sec). The results of the tests are shown in Tables 13 and 14 and Figures 23 to 28. The experiments were conducted filling the height (17.15 cm) of the 5 cm cylinder with sand and gravel. However, three heights of the sand layer were used in the experiments. These heights were equal to 2.5, 5.0, and 10 cm. The rest of the cylinder height was filled by gravel. The results of the experiments indicate that the height of the sand layer influenced the value of the angle of repose regardless of the lifting velocity of the cylinders and the roughness of the bases used (Tables 13 and 14 and Figs. 23 to 28). The laboratory results indicate that the higher the height of the sand layer in the samples, the lower was the angle of repose. An explanation for this result seems to be in the amount of sand in the lower layer. As the amount of sand increases in the lower layer, the sand controls the angle of repose. As the sand layer decrease in thickness, the gravel seems to control the angle of repose. Also, as spreading of the grains takes place, the sand grains which are smaller, allow the gravel grains to rotate during spreading. This explains why the angle of repose of the sample with 10 cm of sand is smaller than the angle of repose measured in the experiment when the 5 cm cylinder was completely filled with sand (Table 5).

Table 13. Cylinder experiment with layered system (sand at the bottom of gravel) with slow velocity (about 2.5cm/sec)

Height of sand at bottom (cm)	Angle of repose on smooth base (degrees)	Angle of repose on rough base (degrees)
2.5	24	25
5.0	21	22
10.0	18.5	20.5

Table 14. Cylinder experiment with layered system (sand at the bottom of gravel) with high velocity (about 7.0cm/sec)

Height of sand at bottom (cm)	Angle of repose on smooth base (degrees)	Angle of repose on rough base (degrees)
2.5	17.1	18.4
5.0	16.1	17.9
10.0	14.2	15.9

Figure 23. Cylinder experiments using layered system on smooth base (2.5cm height sand at bottom), slow velocity

Figure 24. Cylinder experiments using layered system on smooth base (5.0cm height sand at bottom), slow velocity

Figure 25. Cylinder experiments using layered system on smooth base (10.0cm height sand at bottom), slow velocity

Figure 26. Cylinder experiments using layered system on rough base (2.5cm height sand at bottom), slow velocity

Figure 27. Cylinder experiments using layered system on rough base (5.0cm height sand at bottom), slow velocity

Figure 28. Cylinder experiments using layered system on rough base (10.0cm height sand at bottom), slow velocity

7.0 ANALYSIS AND APPLICATION OF THE RESULTS

7.1 MEASURED AND THEORETICAL ANGLE OF REPOSE OF GRANULAR SYSTEMS ON SMOOTH AND ROUGH BASES

The effect of the type of base (rough and smooth) on the angle of repose is analyzed using the laboratory results outlined in Section 6. First, the measured angle of repose obtained by the cylinder tests for the case of sands and gravels when they are released at a slow velocity (2 cm/sec) are compared with the theoretical analysis developed to calculate the angle of repose when the base is rough [Equation (9)], and the case when the base is smooth [Equation (10)]. This has been done in Table 15. An analysis of the results of this table indicates that the degree of roughness of the base on which the granular materials are released influences the values of the angle of repose. The rougher is the base; the higher is the angle of repose. The mechanism how the angle of repose is achieved on the rough and smooth bases is different. The angle of repose attained on the rough base develops when the granular material moves on the face of the granular pile (Fig. 7). The angle of repose on the smooth surface is developed when the granular pile spreads on the smooth surface (Fig. 9). The angle of repose measured in the experiments compared well with the theoretical ones obtained when one uses Equations (9) and (10).

Table 15. Measured and calculated angle of repose on rough and smooth surface

Material	Base Type	Angle of Internal Friction, ϕ (degrees)	Interface Friction Angle, δ (degrees)	Measured Angle of Repose, β , Using Cylinder* (degrees)	Calculated Angle of Repose, β . (degrees)
Sand	Porous Stone	27	25	27	27 **
	Glass Plate	27	16	25	26.5 ***
Gravel	Porous Stone	34	26	33.5	34 **
	Glass Plate	34	15	28	32 ***

* Measured when the lifting velocity of the cylinder was 2 cm/sec

** Calculated using Eq. (9)

*** Calculated using Eq. (10)

The effect of the roughness of the bases was also evaluated using the experiments on mixtures of sand and gravel and on layered systems of gravel and sand described in Section 6 of this thesis. The effect the roughness of the bases on the angle of repose is shown in Figures 29 to 32.

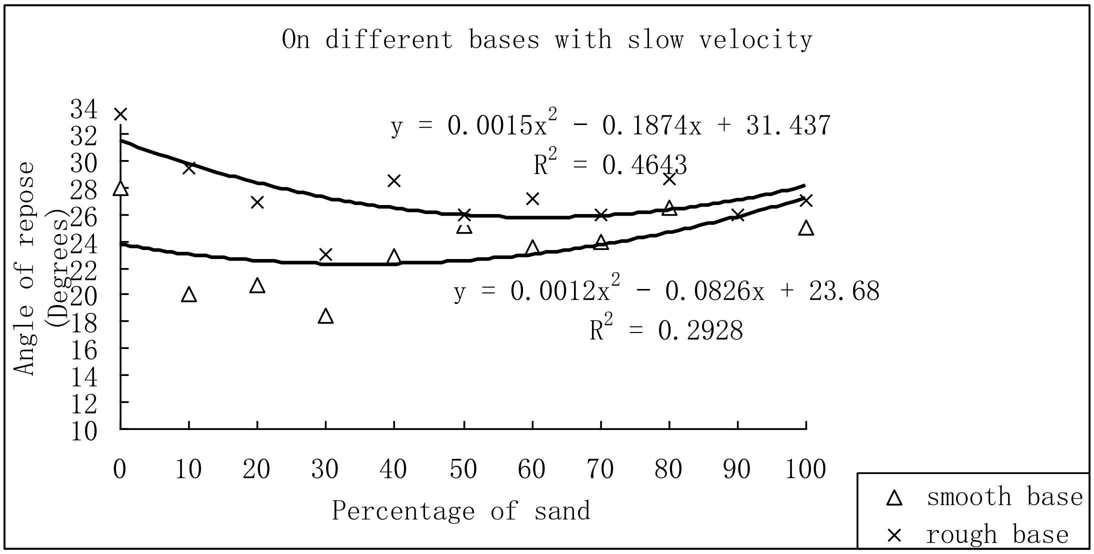


Figure 29. Comparison of results sand-gravel mixture on different bases with slow velocity (data from Table 11)

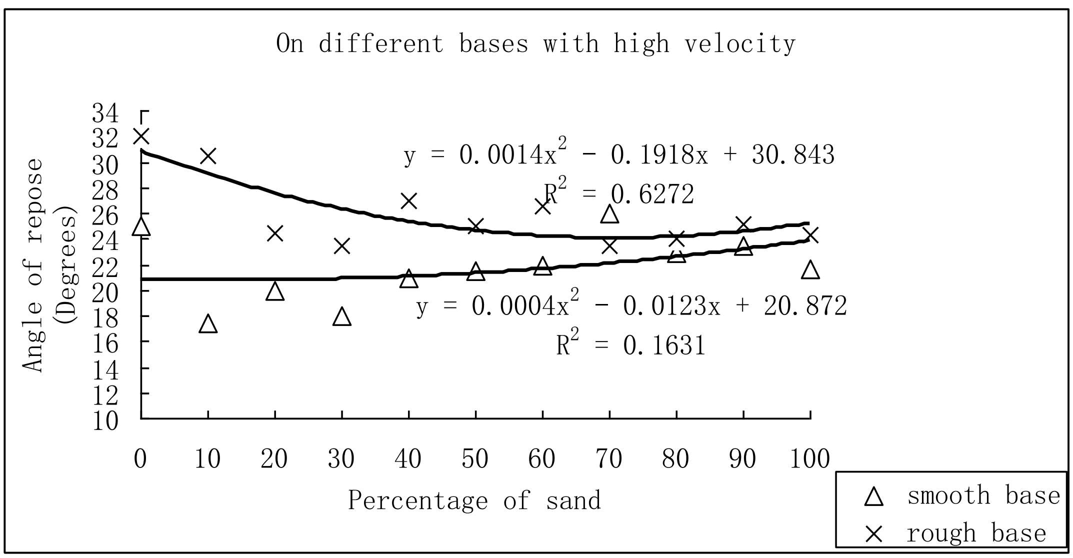


Figure 30. Comparison of results for sand-gravel mixture on different bases with high velocity (data from Table 12)

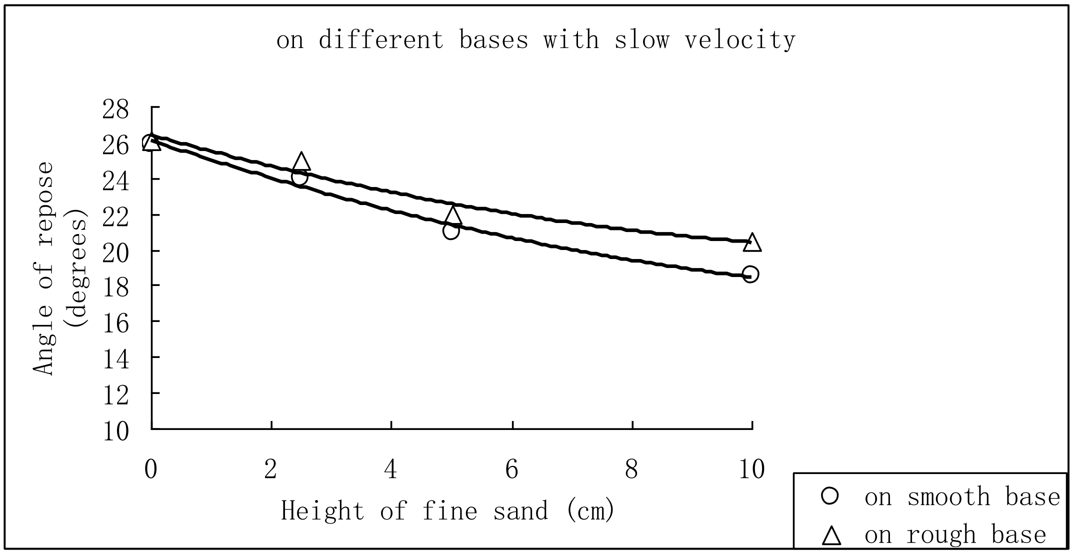


Figure 31. Comparison of results for gravel-sand layered system on different bases with slow velocity (data from Table 13)

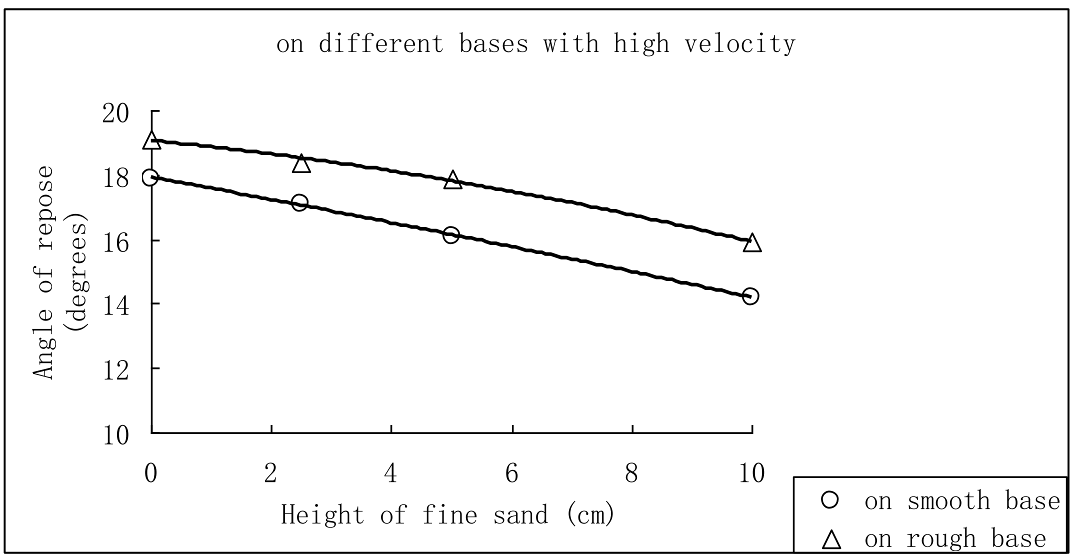


Figure 32. Comparison of results gravel-sand layered system on different bases with high velocity (data from Table 14)

The results for the sand-gravel mixtures shown in Figs. 29 and 30 indicates that the mixtures of sand and gravel developed higher angles of repose when the cylinder tests took place on a base that was rough (porous stone). Also, these figures indicate that when the mixtures were released on a rough surface, the angle of repose decreased with the percentage of sand in the mixtures. However, when the mixtures were released on a smooth surface, there is a tendency for the angle of repose to increase with the percentage of sand in the mixtures. According to Chik and Vallejo (2005), when measuring the angle of repose of a mixture of large (5 mm diameter glass beads) and small (0.4 mm glass beads) glass beads by releasing them from a funnel into a glass plate, they found that the higher the percentage of small beads in the mixture, the higher was the total contact area of the particles with the glass plate. As the contact area between the particles and glass plate increased, the angle of repose increased (Chik and Vallejo, 2005). The same seems to have taken place when testing sand and gravel mixtures on the glass plate.

The results of the angle of repose tests on the layered granular system (gravel on top of sand) are shown in Figures 31 and 32. These figures indicate that. as the thickness of the sand layer increased, the angle of repose decreased in value. These results took place regardless of the velocity of cylinder lifting, and the degree of roughness of the bases used in the experiments. To explain these results, we again used the values of the angle of repose measured using the cylinders and homogeneous samples of either sand or gravel (Table 15). Table 15 shows that the angle of repose measured using the cylinders was smaller for sand than gravel. When testing a layered system of gravel on top and sand at the bottom, at the height and amount of sand increases, the angle of repose will tend to be controlled by the sand. Since sand has a lower angle of repose than gravel, as the sand increases in height, the angle of repose decreases in value. This is what Figures 31 and 32 indicate.

How the sand is located in the granular systems (in a mixture form or in a layered form) seems to also have an effect on the angle of repose. The sand in a layered system (Figs. 31 and 32) has a more marked influence on the angle of repose than when the sand is a mixture form (Figs. 29 and 30).

7.2 EFFECT OF THE VELOCITY OF LIFTING OF CYLINDERS CONTAINING A GRANULAR SYSTEM AND ITS APPLICATION

The effect of the lifting velocity of the cylinders on the angle of repose of granular systems can be explained with the use of Figs. 33 and 34. These figures indicate that as the lifting velocity increased, the angle of repose decreased regardless of the degree of roughness of the bases used and regardless of the height of the sand in the layered system.

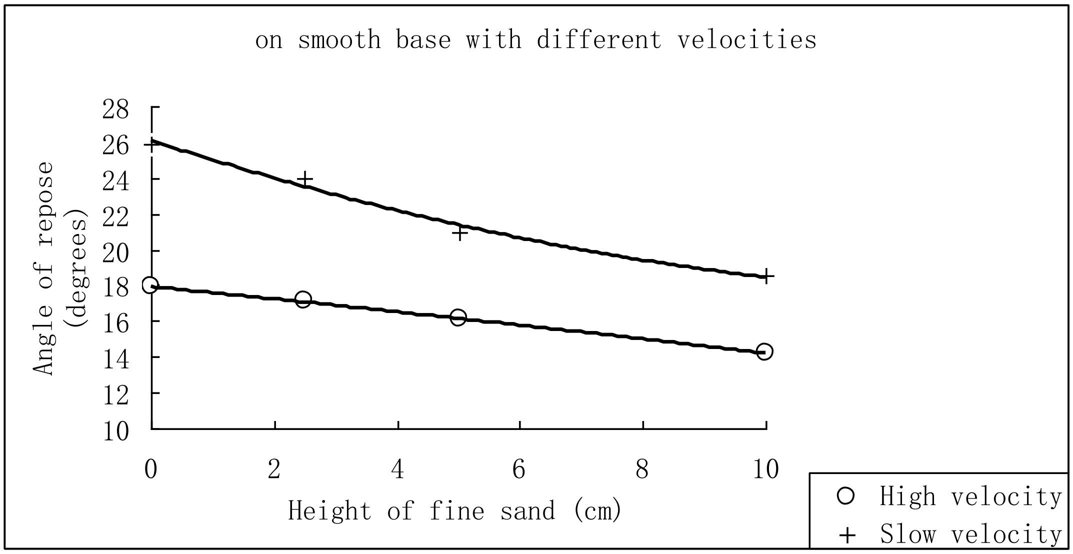


Figure 33. Comparison of results from testing layered system on a smooth base with different lifting velocities of the cylinder (2.5 and 7 cm/sec; Tables 13 and 14)

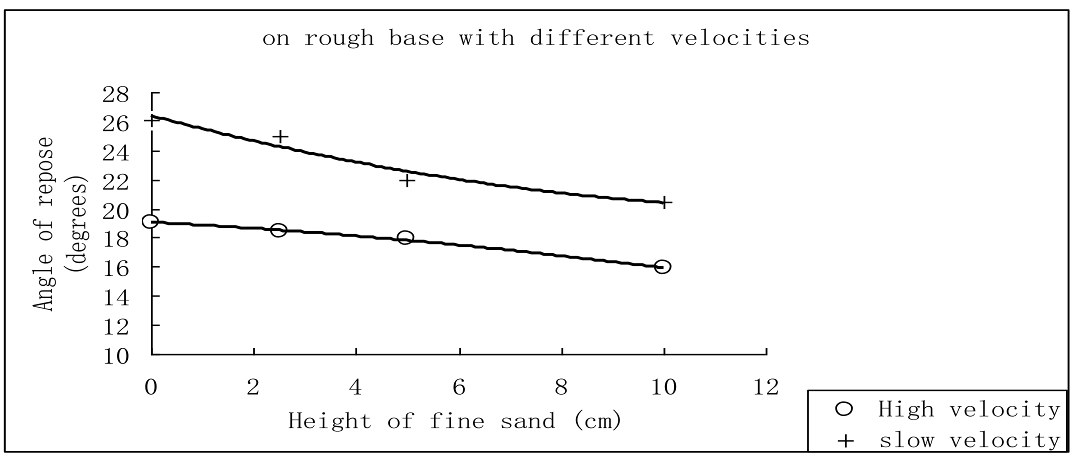


Figure 34. Comparison of results from testing layered system on a rough base with different lifting velocities of the cylinders (2.5 and 7 cm/sec; Tables 13 and 14)

The lifting velocity simulates what happens in the field during a rock failure of a rock slope (Fig. 12). The failure of the steep face could take place either slow or fast. It fails in a slow mode due to weathering. Due to physical weathering (heating and cooling, freezing and thawing), the steep portion of the slope could break slowly and fail at a slow pace after which accumulates slowly at the toe of the slope (Fig. 12). The steep face of the slope could also fail very rapidly due to earthquake forces. The broken pieces of the slope face then will accumulate very rapidly at the toe of the slope. The results of Figures 31 to 34 indicate that the velocity of lifting the cylinders (that simulate the velocity of failure of the steep face of the slope) has an influence on the angle of repose for granular materials. The slower the failure (low lifting velocity of the cylinder), the higher is the angle of repose of granular system (Fig. 12).

7.3 THE EFFECT OF THE WEIGHT OF A HOMOGENEOUS GRANULAR LAYER ON THE ANGLE OF REPOSE

The effect of the weight (height) of the layer of gravel in a cylinder on the angle of repose is shown in Figure 35. This figure is based on the results of Table 10. Figure 35 and table 10 indicates that as the height (weight) of the granular material increased in the 12.5 cm cylinder, the angle of repose decreases in value.

This result will help explain what the angle of repose will be in the rock slope in Figure 12, depending upon the amount of material that fails. Fig. 10 and Table 10 indicates that, as the amount of failing material increases in value, the angle of repose decreases in value. Thus, the angle of repose seems to be influenced by the volume (weight) of material that fails and accumulates at the toe of slopes made of steep section and a flatter accumulated section (Fig. 12). The larger the volumes of the failed material, the smaller will the angle of repose of the deposited material.

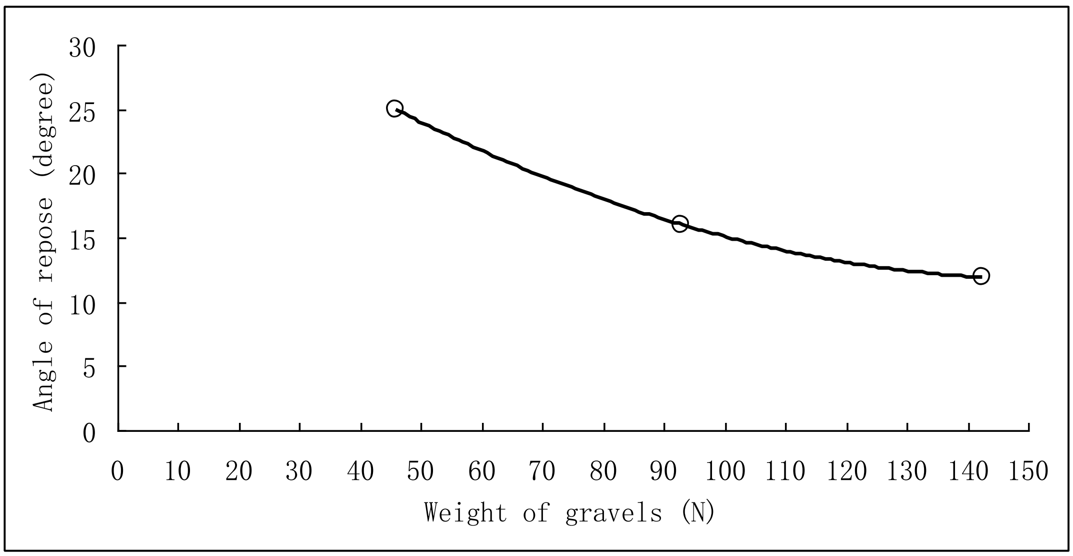


Figure 35. Results of cylinder experiments using gravel with different weights (different heights) on wooden surface and slow lifting velocity (data from Table 10)

8.0 CONCLUSIONS

The angle of repose of granular systems was investigated using hollow cylinders of different diameters and lengths and bases of different degree of roughness. The cylinders contained homogeneous samples of sand and gravel as well composite mixtures of sand and gravel with the sand either thoroughly mixed with gravel or in layered systems. The granular materials developed a conical pile after the cylinders were lifted at two different velocities. From the laboratory experiments, the following conclusions can be made:

(1) The angle of repose of the granular systems was influenced by the degree of roughness of the base on which the grains come to rest. It was determined that the rougher the base was, the higher was the angle of repose.

(2) The mode of failure of the conical pile of grains was different depending if the base was rough or smooth. For the case of a rough base, the initial lifting of the cylinders caused the granular material to form first a pile of conical shape. As the lifting of the cylinders continued, more material was released from the cylinders. This additional material moved on the face of the

initially formed conical shape. The conical shape increased in height maintaining, however, its initial conical shape. For the case of a smooth base, the lifting of the cylinders caused the granular material to form first a pile of conical shape. As the lifting of the cylinders continued, more material was released from the cylinders. This additional material caused the original conical shape to spread and fail over the smooth base. During the spreading, the conical pile of granular material maintained its shape.

(3) The lifting velocities of the cylinders were varied between a slow velocity (2 to 3 cm/sec) and a high velocity (7 to 8 cm/sec). The angle of repose was found to be smaller when the high velocity of cylinder lifting was used regardless of the degree of roughness of the bases. The lifting velocities of the cylinders represent the velocity of failure of a granular material from the top section of a slope of a composite shape (steep top section, semi-flat lower section). Failure of the steep section of a slope by physical degradation is always slow. Failure of the steep section of a slope resulting from seismic loading is always fast.

(4) The angle of repose was also found to decrease in value as the amount of material contained in the cylinders increased in value. This result reflected field findings that indicated that the angle of repose of material at the toe of a composite in shape failed slope decreased in value as the amount (volume) involved in the failure increased in value.

(5) The experiments on mixtures of two type of granular material (sand and gravel) indicated that the angle of repose of these mixtures, when released from the cylinders that contained them, decreased in value as the percentage of sand in the mixture increased in value. It seems that as the sand increased in value, it controlled the angle of repose of the mixtures (the angle of repose of homogeneous sand is smaller than the angle of a homogeneous gravel).

(6) The experiments on layered granular systems (gravel on top of sand) indicated that, regardless of the lifting velocity of the cylinders, the angle of repose of the layered systems decreased in value as the height of the sand layer in the composite increased in value (the composite had a fixed length, the height of the individual sand and gravel layers were made to fluctuate in value). Since sand has a lower angle of repose than gravel, as the sand increased in value in the layered system, the sand controlled the value of the angle of repose of the granular composite.

(7) The results of the tests helped to explain the angle of repose found in rock slopes made of a steep face and a lower section made of granular material that accumulated after the failure of the steep section of the slope.

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